Electromagnetic (I)
1st Midterm Exam.

College of Electronic Technology
Department of Communications

Date: 13/10/2022
Time: 90 min.

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Answer the following questions:

Q1) Consider the following vectors:

$\vec{A} = 2\hat{a}_x - 3\hat{a}_y + \hat{a}_z$, $\vec{B} = 2\hat{a}_x - \hat{a}_y + 3\hat{a}_z$, $\vec{C} = 4\hat{a}_x + 2\hat{a}_y - 2\hat{a}_z$

Find:

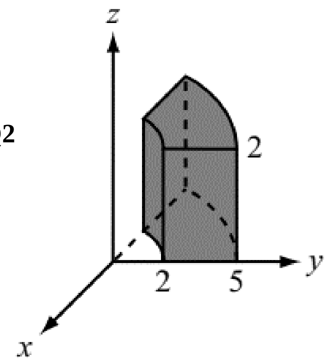
1. $\vec{A}.\vec{B}$
2. $|\vec{A} + 3\vec{B}|$
3. θ_{AB}
4. $Prove\ that$: $\vec{C}$ is perpendicular $\perp$ $to\ both\ \vec{A}\ and\ \vec{B}$

Q2) Determine the volume of a cylindrical surface described by:

$$2m \le \rho \le 5m, \frac{\pi}{2} \le \emptyset \le \pi, 0 \le z \le 2m$$

As in the figure Q2

Figure Q2



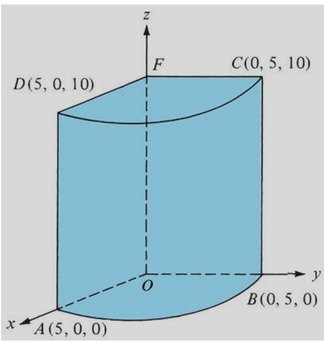
Q3) Transform the vector $\vec{A} = (y - x)\hat{a}_x + (x - y)\hat{a}_y$,From Cartesian to cylindrical coordinates, and evaluate at P$(1, 0^{\circ}, 2)$

Q4) For the vector field $\vec{G} = \rho\hat{a}_\rho + \rho\varphi\ \hat{a}_\varphi + z\hat{a}_z$, Determine the following:

a) The flux of vector $\vec{G}$ through the closed surface shown in figure Q4.

b) Verify Divergence's theorem for the given vector field.

Figure Q4



Vector Relations

Cartesian Coordinates (x,y,z)

$$\boldsymbol{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla\cdot\boldsymbol{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\boldsymbol{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\boxed{\begin{aligned} d\vec{l} &= \vec{a}_x dx + \vec{a}_y dy + \vec{a}_z dz \\ d\vec{l} &= \vec{a}_\rho d\rho + \vec{a}_\phi \rho d\phi + \vec{a}_z dz \\ d\vec{l} &= \vec{a}_r dr + \vec{a}_\theta r d\theta + \vec{a}_\phi r\sin\theta d\phi \end{aligned}}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Cylindrical Coordinates (ρ,ϕ,z)

$$\boldsymbol{A} = A_\rho\mathbf{a}_\rho + A_\phi\mathbf{a}_\phi + A_z\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla\cdot\boldsymbol{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\boldsymbol{A} = \frac{1}{\rho}\begin{vmatrix} \mathbf{a}_\rho & \rho\mathbf{a}_\phi & \mathbf{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix}\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix}\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

$$\boxed{\begin{aligned} &\rho = \sqrt{x^2+y^2}, \quad x = \rho\cos\phi, \\ &\phi = \tan^{-1}\frac{y}{x}, \quad y = \rho\sin\phi, \\ &\qquad\qquad\qquad z = z \\ &z = z \end{aligned}}$$

Spherical Coordinates (r,θ,ϕ)

$$\boldsymbol{A} = A_r\mathbf{a}_r + A_\theta\mathbf{a}_\theta + A_\phi\mathbf{a}_\phi$$

$$\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial\theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}\mathbf{a}_\phi$$

$$\nabla\cdot\boldsymbol{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\boldsymbol{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \mathbf{a}_r & r\mathbf{a}_\theta & (r\sin\theta)\mathbf{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_r & rA_\theta & (r\sin\theta)A_\phi \end{vmatrix}$$

$$\boxed{\begin{aligned} &r = \sqrt{x^2+y^2+z^2} \\ &\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z} \\ &\phi = \tan^{-1}\frac{y}{x} \\ &x = \rho\cos\phi = r\sin\theta\cos\phi \\ &y = \rho\sin\phi = r\sin\theta\sin\phi \\ &z = r\cos\theta \end{aligned}}$$

$$\boxed{\oint_L \mathrm{A}\cdot d\mathrm{l} = \int_S (\nabla\times\mathrm{A})\cdot d\mathrm{S}}$$

$$\boxed{\oint_S \mathrm{A}\cdot d\mathrm{S} = \int_v \nabla\cdot\mathrm{A}\, dv}$$

Q1 :-

① $\vec{A}.\vec{B} = ((2)(2)+(3)(1)+(1)(3)) = 4+3+3 = 10$ 2.5

② $|\vec{A}+3\vec{B}| = |(2+6)\hat{a}_x - (3+3)\hat{a}_y + (1+9)\hat{a}_z|$ 2.5

$= |8\hat{a}_x - 6\hat{a}_y + 10\hat{a}_z| = \sqrt{64+36+100}$

$= \sqrt{200} = 10\sqrt{2}$

③ $\theta_{AB} = \cos^{-1}\left(\frac{\vec{A}.\vec{B}}{|A||B|}\right) = \cos^{-1}\left(\frac{10}{\sqrt{4+9+1}\sqrt{4+1+9}}\right) = \cos^{-1}\left(\frac{10}{\sqrt{14}\sqrt{14}}\right)$ 2

$\theta_{AB} = \cos^{-1}\left(\frac{10}{14}\right) = \cos^{-1}\left(\frac{5}{7}\right) = 44.41°$

$\vec{A}.\vec{C} = 8-6-2 = 0$

$\vec{B}.\vec{C} = 8-2-6 = 0$ 2 2

or $\vec{C}\times(\vec{A}\times\vec{B}) = 0$, $\vec{C}$ is parallel to the surface vector of $\vec{A}\times\vec{B}$

$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 2 & -3 & 1 \\ 2 & -1 & 3 \end{vmatrix} = -8\hat{a}_x - 4\hat{a}_y + 4\hat{a}_z$

4

$\vec{C}\times(\vec{A}\times\vec{B}) = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 4 & 2 & -2 \\ -8 & -4 & 4 \end{vmatrix}$

$= (0)\hat{a}_x - (0)\hat{a}_y + (0)\hat{a}_z$ #

Q_2 :- $V = \oint_V dv = \int_0^2 \int_{\frac{\pi}{2}}^{\pi} \int_2^5 \rho \, d\rho \, d\phi \, dz$

$$V = \int_2^5 \rho \, d\rho \int_{\frac{\pi}{2}}^{\pi} d\phi \int_0^2 dz$$

$$= \frac{\rho^2}{2}\Big|_2^5 \; \phi\Big|_{\frac{\pi}{2}}^{\pi} \; z\Big|_0^2$$

$$= \left(\frac{25-4}{2}\right)\left(\frac{\pi}{\cancel{2}}\right)\cancel{(2)}$$

$$= \frac{21\pi}{2} \, m^3 \simeq 33 \, m^3$$

4

Q_3 :- $\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y-x \\ x-y \\ 0 \end{bmatrix}$

$A_\rho = (y-x)\cos\phi + (x-y)\sin(\phi)$

$A_\phi = -(y-x)\sin\phi + (x-y)\cos\phi$

$A_\rho = \rho(\sin(\phi) - \cos(\phi))\cos(\phi) + \rho(\cos(\phi) - \sin(\phi))\sin(\phi)$

$= \rho\sin(\phi)\cos(\phi) - \rho\cos^2(\phi) + \rho\sin(\phi)\cos(\phi) - \rho\sin^2(\phi)$

$= 2\rho\sin(\phi)\cos(\phi) - \rho(\sin^2(\phi) + \cos^2(\phi))$

$A_\rho = \rho\sin(2\phi) - \rho \quad = \rho(\sin(2\phi) - 1)$

$A\phi = -\rho(\sin(\phi) - \cos(\phi))\sin\phi + \rho(\cos(\phi) - \sin(\phi))\cos(\phi)$

$= -\rho\sin^2(\phi) + \cancel{\rho\sin(\phi)\cos(\phi)} - \cancel{\rho\sin(\phi)\cos(\phi)} + \rho\cos^2(\phi)$

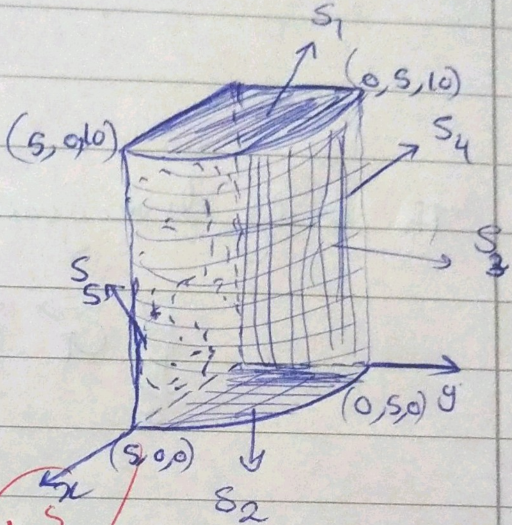
$= \rho(\cos^2(\phi) - \sin^2(\phi)) = \rho\cos(2\phi)$

$\vec{A} = \rho(\sin 2\phi - 1)\hat{a}_\rho + \rho \cos(2\phi)\, a_\phi$ 6

At $P(1, 0^\circ, 2) \Rightarrow \vec{A} = 1(\sin(0) - 1)\hat{a}_\rho + 1\cos(0)\, a_\phi$

$$\vec{A} = -\hat{a}_\rho + \hat{a}_\phi$$

Q4 $\vec{G} = \rho \hat{a}_\rho + \rho\phi \hat{a}_\phi + z \hat{a}_z$



For $S_1 \Rightarrow z = 10 \Rightarrow \vec{ds} = \rho\, d\rho\, d\phi\, \hat{a}_z$

$$\psi_1 = \int_0^{\frac{\pi}{2}}\int_0^{5} \rho z\, d\rho\, d\phi$$

$$\psi = z\, \frac{\rho^2}{2}\Big|_0^5\ \phi\Big|_0^{\frac{\pi}{2}} = 10\left(\frac{25}{2}\right)\left(\frac{\pi}{2}\right)$$

$$= \frac{125}{2}\pi$$ 1.5

For $S_2 \Rightarrow z = 0 \Rightarrow \vec{ds} = \rho\, d\rho\, d\phi\, \hat{a}_z$

zero

$$\psi_2 = \int_0^{\frac{\pi}{2}}\int_0^{5} \rho (z)\, d\rho\, d\phi = 0$$ 1.5

For $S_3 \Rightarrow \rho = 5,\ \vec{ds} = \rho\, d\phi\, dz\, \hat{a}_\rho$

$$\psi_3 = \rho^2 \int_0^{10}\int_0^{\frac{\pi}{2}} d\phi\, dz = 25\ z\Big|_0^{10}\ \phi\Big|_0^{\frac{\pi}{2}} = 25(10)\left(\frac{\pi}{2}\right) = \frac{250\pi}{2}$$ 1.5

For $S_4 \Rightarrow \phi = \frac{\pi}{2},\ \vec{ds} = d\rho\, dz\, \hat{a}_\phi$

$$\psi_4 = \phi \int_0^{10}\int_0^{5} \rho\, d\rho\, dz = \frac{\pi}{2}\ \frac{\rho^2}{2}\Big|_0^5\ z\Big|_0^{10} = \frac{\pi}{2}\left(\frac{25}{2}\right)(10) = \frac{125}{2}\pi$$ 1.5

For $S_5 \Rightarrow \phi = 0 \Rightarrow \vec{ds} = -d\rho\, d_z\, \hat{a}_\phi$

$$\psi_5 = \underbrace{\phi}_{=\text{zero}} \int_0^{10}\int_0^{5} \rho\, d\rho\, d_z = 0$$

1.5

$$\oiint_S \vec{G}\cdot\vec{ds} = \psi_1 + \psi_2 + \psi_3 + \psi_4 + \psi_5 = \frac{125}{2}\pi + 0 + \frac{250}{2}\pi + \frac{125}{2}\pi + 0$$

$$= 250\pi$$

(b) To verify divergence theorem

$$\oiint_S \vec{G}\cdot\vec{ds} = \int_V \nabla\cdot\vec{G}\, dv$$

2.5

$$\nabla\cdot\vec{G} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho^2) + \frac{1}{\rho}\frac{\partial}{\partial\phi}(\rho\phi) + \frac{\partial}{\partial z}(z)$$

$$= \frac{1}{\cancel{\rho}}(2\cancel{\rho}) + \frac{1}{\cancel{\rho}}(\cancel{\rho}) + 1 = \underline{\underline{4}}$$

$$\int_V \nabla\cdot\vec{G}\, dv = 4\int_0^{10}\int_0^{\frac{\pi}{2}}\int_0^{5} \rho\, d\rho\, d\phi\, d_z$$

$$= 4\left.\frac{\rho^2}{2}\right|_0^5 \left.\phi\right|_0^{\frac{\pi}{2}} \left.z\right|_0^{10} = 4\left(\frac{25}{\cancel{2}}\right)\left(\frac{\pi}{\cancel{2}}\right)(10)$$

$$= 250\pi$$

$=$